# Wave Dynamics of ψ — Klein-Gordon Evolution and Oscillating Gravity

## 🎯 Objective

To simulate the full Klein-Gordon equation for the ψ field, allowing it to propagate and oscillate like a wave field. This represents the next level of realism where ψ behaves as a physical-like field, modulated by potential energy (via ).  
We observe how oscillations in ψ create evolving gravitational patterns via curvature coupling.

## 🧾 Key Equations

### 🔹 Klein-Gordon Equation for ψ

We evolve ψ using the second-order field equation:

Plaintext:  
d²ψ/dt² - Laplacian(ψ) + mψ² \* ψ = 0

### 🔹 Discrete Update Equation

Implemented as a finite difference scheme:

Plaintext:  
psi[t+1] = 2*psi[t] - psi[t-1] + dt****2 \* (laplacian(psi[t]) - mψ****2*  psi[t])

### 🔹 Gravity from Evolving ψ

With static curvature from :

Plaintext:  
gravity[t] = curvature \* psi[t]

## 📈 Plot: ψ(x, t) Wave Dynamics

* Initial ψ is a Gaussian bump
* ψ spreads and forms wavefronts, propagating left and right
* Small oscillations form behind the wavefront, typical of Klein-Gordon behavior
* Amplitude is modulated by mass term

## 📉 Plot: Gravity(x, t)

* Gravity tracks ψ closely since curvature is static
* The gravity field oscillates and migrates with ψ
* Strongest gravity occurs where ψ remains nonzero and curvature is nontrivial
* Clear spatial “ripples” form: a gravitational wave-like structure

## 🧠 Interpretation

* ψ now exhibits propagation, oscillation, and decay
* Mass term prevents runaway solutions; it acts like a spring restoring force
* Gravity becomes a dynamic field, reflecting ψ’s shifting energy

## 🧬 Physical Insights

* ψ is no longer a static modifier — it now behaves as a vibrating generative field
* Gravity appears as a modulated projection of ψ, scaled by curvature
* Could produce gravitational wave pulses if curvature is dynamic in future steps

## 📐 Diagram: Dynamic Loop with Oscillation

ψ(x, t)  
 ↓

d²ψ/dt² ← mψ² ψ + ∇²ψ

↓

Varying ψ(x, t)

↓

Gravity(x, t) = Curvature × ψ(x, t)

## 🧱 Simulation Summary

| Feature | Description |
| --- | --- |
| Grid | 100 spatial points |
| Steps | 100 time steps |
| ψ₀(x) | Gaussian center bump |
| mψ | 0.5 |
| dt | 0.1 |
| dx | 1.0 |
| Boundary | Zero Dirichlet (fixed ends) |

## 🌀 Emergent Patterns

* Wave splitting: ψ moves in both directions
* Localized gravity zones: peaks in ψ create peaks in gravity
* Oscillatory structure: mimics gravitational pulses (early prediction of GWs)

## 🔎 Next in Phase 5 – Part 4

We will:

* Add potential term into the feedback loop
* Explore nonlinear potentials:

Plaintext:  
V(psi) = 0.5 \* m\_psi^2 \* psi^2 + 0.25 \* λ \* psi^4

* Study:
  + Standing waves
  + Merging ψ wells
  + Field interference
* Evaluate total energy density over space and time
* Look for phase transitions in ψ